Global Value Numbering

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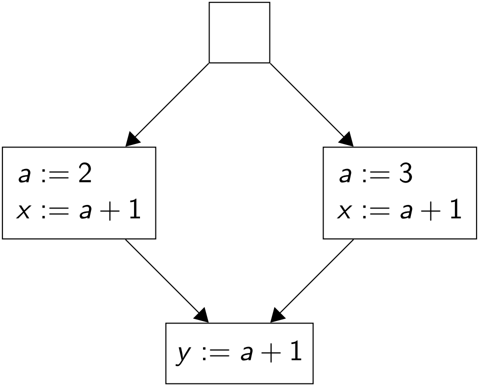
SAARLAND UNIVERSITY
COMPUTER SCIENCE

Value Numbering



- Replace second computation of $a + 1$ with a copy from x

Value Numbering

- Goal: Eliminate redundant computations
- Find out if two variables have the same value at given program point
  - In general undecidable
- Potentially replace computation of latter variable with contents of the former
- Resort to Herbrand equivalence:
  - Do not consider the interpretation of operators
  - Two expressions are equal if they are structurally equal
- This lecture: A costly program analysis which finds all Herbrand equivalences in a program and a "light-weight" version that is often used in practice.

Herbrand Interpretation

- The Herbrand interpretation $\mathcal{I}$ of an n-ary operator ω is given as

$$\mathcal{I}(\omega) : T^n \to T \qquad \mathcal{I}(\omega)(t_1, \ldots, t_n) := \omega(t_1, \ldots, t_n)$$

  Especially, constants are mapped to themselves

- With a state σ that maps variables to terms

$$\sigma : V \to T$$

- we can define the Herbrand semantics $\langle t \rangle \sigma$ of a term t

$$\langle t \rangle \sigma := \begin{cases} \sigma(v) & \text{if } t = v \text{ is a variable} \\ \mathcal{I}(\omega)(\langle x_1 \rangle \sigma, \ldots, \langle x_n \rangle \sigma) & \text{if } t = \omega(x_1, \ldots, x_n) \end{cases}$$

Programs with Herbrand Semantics

- We now interpret the program with respect to the Herbrand semantics
- For an assignment

$$x \leftarrow t$$

  the semantics is defined by:

$$[\![x \leftarrow t]\!]\sigma := \sigma\,[\langle t\rangle\sigma / x]$$

- The state after executing a path $p : \ell_1, \ldots, \ell_n$ starting with state σ_0 is then:

$$[\![p]\!]\sigma_0 := ([\![\ell_n]\!] \circ \cdots \circ [\![\ell_1]\!])\sigma_0$$

- Two expressions t_1 and t_2 are Herbrand equivalent at a program point ℓ iff

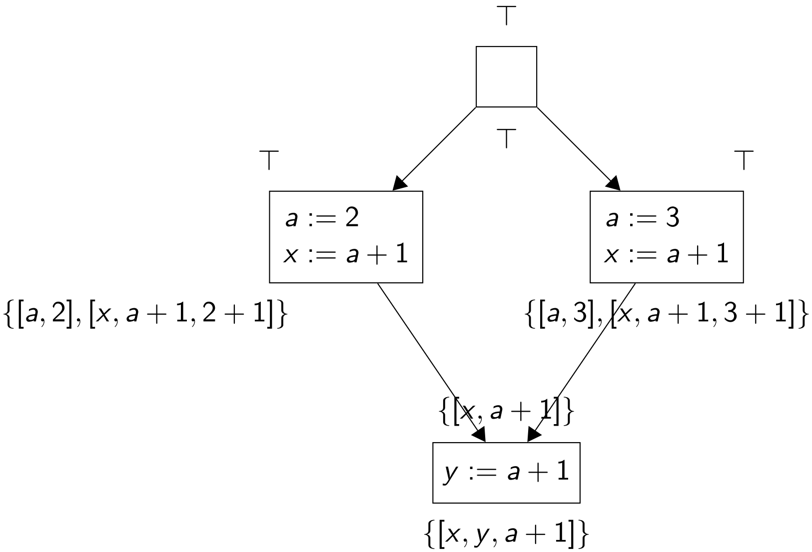
$$\forall p : r, \ldots, \ell.\, \langle t_1\rangle[\![p]\!]\sigma_0 = \langle t_2\rangle[\![p]\!]\sigma_0$$

Kildall's Analysis

- Track Herbrand equivalences with a forward data flow analysis
- A lattice element is an equivalence class of the terms and variables of the program
- The equivalence relation is a congruence relation w.r.t. to the operators in our expression language.
  For each operator ω, each eq. relation R, and $e, e_1, \dots \in V \cup T$:
  $$e\; R\; (e_1\; \omega\; e_2) \implies e_1\; R\; e_1' \implies e_2\; R\; e_2' \implies e\; R\; (e_1'\; \omega\; e_2')$$
- Two equivalence classes are joined by intersecting them
  $R \sqcup S := R \cap S := \{(a, b) \mid a\; R\; b \land a\; S\; b\}$
- $\perp = \{(x, y) \mid x, y \in V \cup T\}$
  ☞ optimistically assume all variables/terms are equivalent
- Initialize with $\top = \{(x, x) \mid x \in V \cup T\}$
  ☞ at the beginning, nothing is equivalent

Kildall's Analysis

Example



Kildall's Analysis

Transfer Functions

... of an assignment

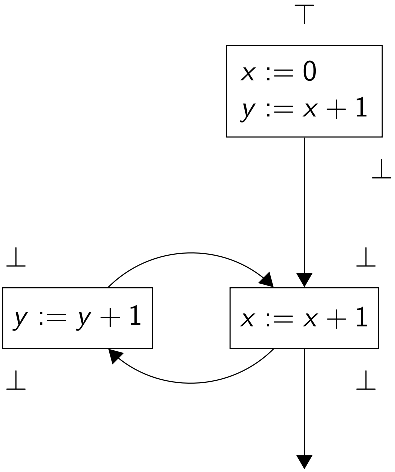
$$\ell : x \leftarrow t$$

- Compute a new partition checking (in the old partition) who is equivalent if we replace x by t

$$[\![x \leftarrow t]\!]^\sharp R := \{(t_1, t_2) \mid t_1[t/x] \; R \; t_2[t/x]\}$$

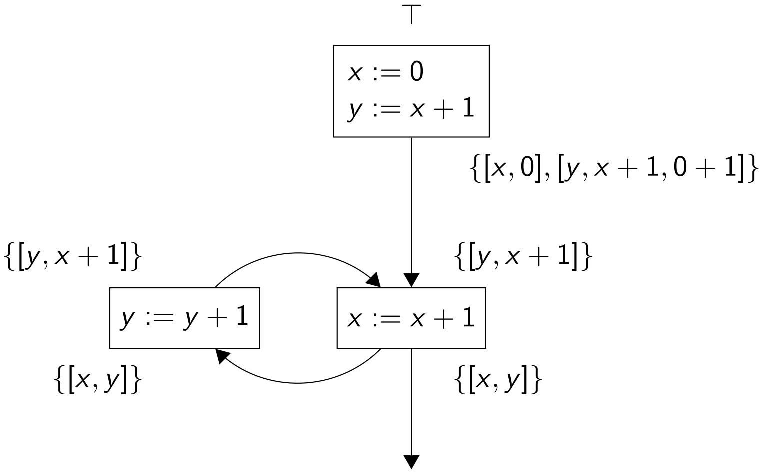
Kildall's Analysis

Example



Kildall's Analysis

Example



Kildall's Analysis

Comments

- One can show that Kildall's Analysis is sound and complete
- Naïve implementations suffer from exponential explosion (pathological):
  - Because the equivalence relation must be a congruence size of eq. classes can explode:

$$R = \{[a, b], [c, d], [e, f], [x, a + c, a + d, b + c, b + d], \\ [y, x + e, x + f, (a + c) + e, \ldots, (b + d) + f]\}$$

- In practice: Do not make congruence explicit in representation
- Instead: Before analysis, scan program for all appearing expressions (and subexpressions!) and only include those in the representation of the equivalence classes

The Alpern, Wegman, Zadeck (AWZ) Algorithm

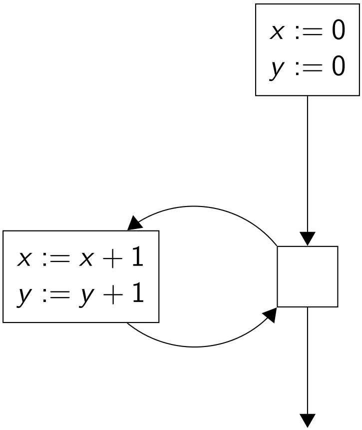
- Incomplete
- Flow-insensitive
  - does not compute the equivalences for every program point but sound equivalences for the whole program
- Uses SSA
  - Control-flow joins are represented by ϕs
  - Treat ϕs like every other operator (cause for incompleteness)
  - SSA compensates flow-insensitivity
- Interpret the SSA data dependence graph as a finite automaton and minimize it
  - Refine partitions of "equivalent states"
  - Using Hopcroft's algorithm, this can be done in $O(e \cdot \log e)$

The AWZ Algorithm

- In contrast to finite automata, do not create two partitions but a class for every operator symbol
  - Note that the ϕ's block is part of the operator
  - Two ϕs from different blocks have to be in different classes
- Optimistically place all nodes with the same operator symbol in the same class
  - Finds the least fixpoint
  - You can also start with singleton classes and merge but this will (in general) not give the least fixpoint
- Successively split class when two nodes in the class are detected not equivalent

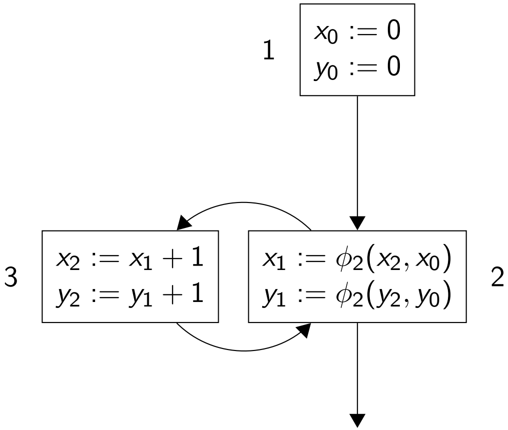
The AWZ Algorithm

Example



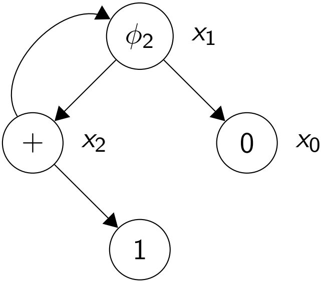
The AWZ Algorithm

Example

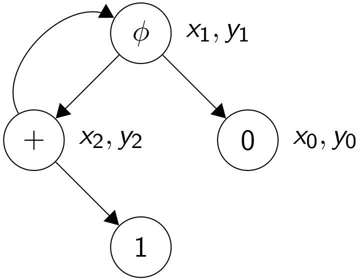


The AWZ Algorithm

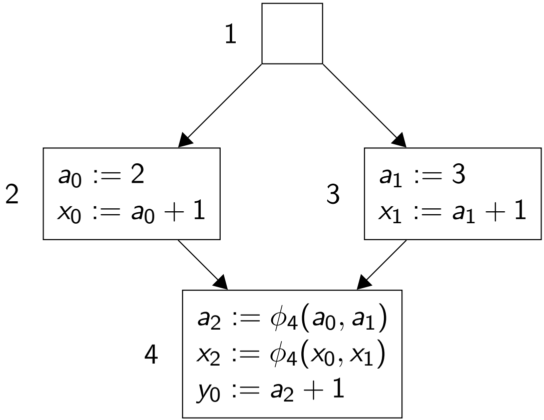
Example



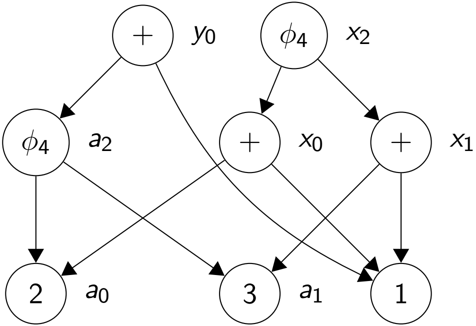
The AWZ Algorithm
Example



Kildall compared to AWZ



Kildall compared to AWZ



Kildall compared to AWZ